

An extensive study of dynamical friction in dwarf galaxies: the role of stars, dark matter, halo profiles and MOND

F. J. Sánchez-Salcedo[☆], Jorge Reyes-Iturbide and X. Hernandez

Instituto de Astronomía, UNAM, Ciudad Universitaria, Apt. Postal 70 264, C.P. 04510, Mexico City, Mexico

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ABSTRACT

We investigate the in-spiraling timescales of globular clusters in dwarf spheroidal (dSph) and dwarf elliptical (dE) galaxies, due to dynamical friction. We address the problem of these timescales having been variously estimated in the literature as much shorter than a Hubble time. Using self-consistent two-component (dark matter and stars) models, we explore mechanisms which may yield extended dynamical friction timescales in such systems in order to explain why dwarf galaxies often show globular cluster systems. As a general rule, dark matter and stars both give a comparable contribution to the dynamical drag. By exploring various possibilities for their gravitational make-up, it is shown that these studies help constrain the parameters of the dark matter haloes in these galaxies, as well as to test alternatives to dark matter. Under the assumption of a dark halo having a central density core with a typical King core radius somewhat larger than the observed stellar core radius, dynamical friction timescales are naturally extended upwards of a Hubble time. Cuspy dark haloes yield timescales $\lesssim 4.5$ Gyr, for any dark halo parameters in accordance with observations of stellar line-of-sight velocity dispersion in dwarf spheroidal galaxies. We confirm, after a detailed formulation of the dynamical friction problem under the alternative hypothesis of MOND dynamics and in the lack of any dark matter, that due to the enhanced dynamical drag of the stars, the dynamical friction timescales in MOND would be extremely short. Taking the well-measured structural parameters of the Fornax dSph and its globular cluster system as a case study, we conclude that requiring dynamical friction timescales comparable to the Hubble time strongly favours dark haloes with a central core.

Key words: galaxies: dwarf – galaxies: individual (Fornax) – galaxies: kinematics and dynamics – globular clusters: general – gravitation

1 INTRODUCTION

The dynamical friction (DF) timescale for globular clusters (GCs) to sink to the centre of most of dwarf spheroidals (dSph) and dwarf elliptical (dE) galaxies yields a fraction of the Hubble time, when dark halo parameters are taken straight from measurements of the stellar distributions (e.g., Tremaine 1976; Hernandez & Gilmore 1998b; Oh, Lin & Richer 2000; Lotz et al. 2001). Both the stellar bulk of the dwarf galaxy and the dark matter contribute to the DF. In some dE, the DF timescale with the dark matter alone is shorter than one Hubble time, in some dSph, the dynamical friction due to the stars alone would suffice to decay the GC orbits in a few Gyr, under the above assumption. Tremaine (1976) first noticed that using the preliminary values for the radius and mass of the second most luminous of the 10 dSph satellites of the Milky Way, Fornax, this time is $\sim 1\text{--}2$ Gyr, very short as compared to absolute ages estimated for these clusters ($\sim 14.6 \pm 1.0$ Gyr for clusters 1–3 and 5, ~ 11.6 Gyr for cluster 4, see Buonanno et al. 1998 and Mackey & Gilmore 2003). One would expect the formation of bright nuclei from the merger of orbitally decayed GCs. Paradoxically, Fornax contains five GCs, an unusually high GC

[☆] E-mail:jsanchez@astroscu.unam.mx

frequency for its dynamical mass, and does not present a central nucleus. Although one of the clusters is observed located near the centre, it presents a radial velocity comparable to the mean velocity of the field stars (Dubath et al. 1992). Hence, this globular cluster has not been really dragged to the Fornax centre through dynamical friction.

Lotz et al. (2001) found that the nuclei in $M_V > -14$ dE's are several magnitudes fainter than expected[†]. Indeed, there are dE with faint nuclei and high globular cluster specific frequencies. Therefore, either the DF timescale is being underestimated or, in these galaxies, some mechanisms are working against DF to prevent the GC system from collapsing into a bright nucleus (see Lotz et al. 2001 for a discussion of the different possibilities).

In this paper we reconsider the problem of the orbital decay of globular clusters in dwarf galaxies. In the case of central cluster galaxies, a variety of processes may contribute to altering the kinematics of GCs which are beyond the scope of this paper. Using Fornax as a case study, we examine the orbital evolution of GCs in both the conventional dark matter scenario and under MOdified Newtonian Dynamics (MOND; Milgrom 1983). Although our work is aimed at exploring possible explanations for the survival of GCs in dSph and dE, our analysis will be focused mainly on the Fornax dSph because the recent detailed determinations of the structural and dynamical parameters of both Fornax and its GC population (i.e. position, age, mass and radial velocity of the GCs are known), allow us to test our assumptions. Fornax is the prototypical dSph with a high GC frequency, has no nucleus, and short DF timescale estimates.

Since DF is a gravitational effect, our analysis is relevant for any massive perturber, such as black holes, star clusters or any kinematically cold massive substructure (e.g., Kleyna et al. 2004) and not only for GCs. Studies of the survival of GC systems and density substructure can provide useful constraints on the dark halo profile in dSph and dE (e.g., Hernandez & Gilmore 1998b), and could shed light on the debate of cuspy/cored haloes, which is currently a test for the standard Λ CDM paradigm. On the other hand, since modified Newtonian dynamics (MOND; Milgrom 1983) can explain the dynamics of spiral galaxies without any dark matter (e.g., Sanders 1996; McGaugh & de Blok 1998), and perhaps the dynamics of dSph (Lokas 2001, 2002), one could ask whether MOND offers a more natural solution to the problem of the rapid orbital evolution of the globular clusters in dwarf galaxies. Since MOND is solely determined by the luminous material, the DF timescale can be used as a test of modified dynamics.

In §2 we give a general statement of the problem, reviewing estimates of the DF timescales for the GCs in dwarf galaxies, and discussing the processes that may be working against the orbital decay of GC systems and their associated difficulties. Section 3 presents a solution to the dynamical-friction problem in a self-consistent two-component dynamical system, dark halo and stars, giving analytical approximate solutions and full numerical integrations for a range of possible dark halo profiles. In §4 we explore the resulting in-spiraling timescales in the MOND scenario, for various relevant limits of such theory. We find that the problem of the sedimentation of GCs has a simple solution in the dark matter scenario, whereas it appears problematic in the MOND scenario. Our conclusions are summarized in §5.

2 DYNAMICAL FRICITION IN DWARF GALAXIES. STATEMENT OF THE PROBLEM

In the standard Newtonian problem, the deceleration felt by a massive perturber, e.g. a globular cluster, of mass M_p moving at velocity \vec{V} , produced by background particles of mass $m \ll M_p$ and having an isotropic distribution function $f(v)$ is given by

$$\frac{d\vec{V}}{dt} = -16\pi^2 \ln \Lambda G^2 m M_p \frac{\vec{V}}{V^3} \int_0^V f(v) v^2 dv, \quad (1)$$

where $\ln \Lambda$ is the Coulomb logarithm, with $\Lambda \sim b_{\max}/b_{\min}$ and b_{\max} and b_{\min} a maximum and minimum impact parameters relevant to the problem. For an extended perturber with internal velocity dispersion σ_p , the minimum impact parameter is taken as $b_{\min} \approx GM_p/\sigma_p^2$. In the case of a GC moving on a circular orbit in centrifugal equilibrium within the core radius $r_0 \equiv (9\sigma^2/4\pi G\rho_0)^{1/2}$ of a dwarf galaxy, where σ is the one-dimensional velocity dispersion of field particles and ρ_0 the central density, the characteristic orbital decay timescale, derived in the local approximation, is

$$t_{\text{df}} = \frac{1}{\sqrt{3} \ln \Lambda} \left(\frac{\sigma}{1 \text{ km s}^{-1}} \right) \left(\frac{r_0}{1 \text{ kpc}} \right)^2 \left(\frac{M_p}{10^5 M_\odot} \right)^{-1} \text{ Gyr}, \quad (2)$$

(e.g., Tremaine 1976; Hernandez & Gilmore 1998b; Oh et al. 2000).

In Fornax, the observed stellar velocity is $\sigma \approx 11 \text{ km s}^{-1}$ and the luminous core radius $r_0 = 15'$, equivalent to almost $\sim 0.6 \text{ kpc}$ for an adopted distance of 138 kpc (Mateo 1993; Walcher et al. 2003). Adopting $b_{\max} = r_0$ and $b_{\min} = GM_p/4\sigma_p^2$ (e.g., White 1976), $\ln \Lambda \approx 3-4$ for a typical Fornax GC of mass $2 \times 10^5 M_\odot$ and central velocity dispersion 7 km s^{-1} (Dubath et al. 1992). In order to give estimates, we will adopt a value $\ln \Lambda = 3$. Therefore, $t_{\text{df}} \simeq 0.5 \text{ Gyr}$ for such a GC. Since for eccentric orbits with the same energy t_{df} is even shorter (e.g., Lacey & Cole 1993; Colpi et al. 1999), it would appear that

[†] Note that Fornax has a magnitude $M_V = -12.5$.

clusters are expected to sink toward the nucleus of the galaxy due to the dynamical friction with the underlying stellar population (Tremaine 1976; Oh, Lin & Richer 2000). Contrary to this prediction, Fornax possesses five GCs of 14.6 Gyr of age (except cluster 4 which has an age of 11.6 Gyr) at $(1.60, 1.05, 0.43, 0.24, 1.43)$ kpc from the optical centre and masses $(0.37, 1.82, 3.63, 1.32, 1.78) \times 10^5 M_\odot$, respectively (Mackey & Gilmore 2003), but no nucleus. The unusually high globular cluster frequency for its present mass suggests that it was able to preserve its GC system intact without coagulation of GCs in its centre.

Tremaine (1976) proposed that once GCs reach the centre of the galaxy they are ejected by the slingshot instability in a three-body system. However, this expelling mechanism is only effective for point objects. Decayed GCs should all merge together forming a bright nucleus.

Oh, Lin & Richer (2000) have attempted to identify the tidal fields as a possible heating mechanism for GCs in dwarf galaxies. In clusters of galaxies, the tidal interaction between the dE and the tidal field of the entire cluster of galaxies may be one possible stirring source. In the case of Fornax, the tidal influence of the Milky Way may inject kinetic energy into the dispersive motion of the GC population. Indeed, Mayer et al. (2001) have shown that the effects of the Galactic tidal field upon the local satellites can transform an extended low surface brightness dwarf into a dSph.

However, Oh et al. (2000) show that the Galactic tidal effect by itself cannot counter the dynamical-friction drag within Fornax because the latter process operates on a shorter timescale. Only if Fornax has experienced continuous and significant mass loss (50–90%) would the clusters be stirred to their actual distances. This scenario would imply that Fornax is undergoing tidal disintegration and, consequently, a stream of tidally stripped material, asymmetries in the shape of density profile and in its stellar population, are expected. Since Fornax is at perigalactic, as dynamical studies strongly suggest, we should be observing the maximum effect of the tidal interaction right now. However, new wide field data reveals that Fornax has an almost perfect relaxed spherical King profile, and no direct sign of interaction with the tidal field of the Milky Way can be seen (Walcher et al. 2003). The absence of any evidence of tidal stripping is in good agreement with recent numerical work which suggests that tidal stripping and shocking interior to ~ 1 kpc are both negligible for dSph galaxies with orbital pericentre $\gtrsim 30$ kpc and masses $\gtrsim 10^8 M_\odot$ (Read et al. 2006).

As remarked by Read et al. (2006), the result that Local Group dSph have not been significantly stripped interior to 1 kpc needs not invalidate the tidal transformation model presented by Mayer et al. (2001). Indeed, once the galaxy has been transformed into a dSph, the Galactic tidal field ceases to significantly affect the internal dynamics of the dwarf. These would hence result in marginal internal tidal heating, as suggested by the relaxed kinematics and spherical present morphology of Fornax.

To summarize this point, we can say that although Galactic tidal effects upon Fornax can not be completely ruled out, particularly over its poorly known initial stages, tidal heating appears insufficient to account for the survival of the observed GC system.

There are other potential sources of external dynamical heating of the GC population, such as the tidal interaction with a hypothetical companion galaxy that finally merged with the dwarf. Interestingly, there is some evidence of a merger of a small (10^5 – $10^6 M_\odot$) companion that occurred ~ 2 Gyr ago (Coleman et al. 2004). However, a system of that mass would be unable to tidally stir the GC population to the required degree. Another possible mechanism for injecting kinetic energy into the GCs is by scattering of the GCs by massive black holes (Oh et al. 2000). However, the gravitational encounters would disrupt the clusters (e.g., Lotz et al. 2001). Moreover, this scenario predicts some features that are not observed (Oh et al. 2000). Since the drag depends on the mass of the GCs, one could suggest that the GCs formed only recently by successive mergers of less massive GCs. Current observations indicate that the clusters in Fornax are not likely merged objects.

A similar analysis for the GCs in NGC 185 (Tremaine 1976) and dE (Lotz et al. 2001) leads to the same conclusion: if the GCs formed with the same radial profile as the stellar component, the entire GC system would collapse into a single bright nucleus. It is indeed unlikely that all the GCs have been captured recently by the parent dE or dSph or that they constitute a young population, so that the dynamical-friction drag had no time to spiral them into the centre. For a more detailed discussion of the difficulties of the above scenarios, we refer the reader to Oh et al. (2000) and Lotz et al. (2001). In conclusion, the extended distributions of GCs in Fornax and dE's need an explanation.

Notice that in going from Eq. (1) to Eq. (2) it has been assumed that the circular velocity of the object experiencing the dynamical friction is in equilibrium with the potential produced by the same distribution of background field particles $f(v)$ which cause the deceleration. Hence, the core radius of Eq. (2) should be that of the density distribution responsible for the observed σ . The estimates referred to at the start of this section however, consider only one dynamical component, inconsistent in the absence of a self-consistent gravitational model, as the large M/L values of these systems signal. As it will become clear in the following sections, a consideration of the dynamical friction problem in a two-component self-consistent model allows to reconcile the observed dwarf galactic parameters with the expectation of long DF timescales for GCs in these systems.

3 EXAMINING THE TWO-COMPONENT DYNAMICAL FRICTION

Equation (2) provides the DF timescale of a body orbiting within the core of a dSph due to the combined action of field stars and dark matter but only under the assumption that mass-follows-light. We know, however, that dark matter and stars may have different distribution functions; on no galactic scales does mass follow light. In the case of Fornax, it is clear that a single component King model is invalid (e.g., Walker et al. 2005). Hence the use of Eq. (2) is not justified. As suggested by Hernandez & Gilmore (1998b), an increase in the dark matter core will increase the velocity dispersion of the dark matter and decrease the DF with these particles (but not with the stars). It would be desirable to explore if there exists a reasonable range of halo parameters, consistent with the observed stellar projected density and kinematics, for which the dynamical friction is significantly suppressed.

As the DF force depends on the distribution function, we would need self-consistent distribution functions for the stars and dark matter particles moving in the same underlying matter potential. The usual way to proceed is to suppose a form for the distribution functions and constrain the parameters of the model from the measured light distribution and the velocity dispersion profile of the stellar component. Because the King model is known to provide a good fit to the present light distribution of the inner regions of certain dSph and galactic dark haloes (Hernandez & Gilmore 1998a), a two-component King model could be a natural choice (e.g., Pryor & Kormendy 1990).

If $E = -v^2/2 - \phi$ is the total (positive) energy per unit of mass and v the particle velocity, each component j is assumed to have the energy distribution function:

$$f_j(E) \propto e^{-\mu_j(E-v_0^2 W_0)/v_0^2} - 1, \quad (3)$$

where v_0 sets the velocity scale and W_0 is the central depth of the potential in units of v_0^2 . For luminous (subscript \star) and dark (subscript dm) components, the model has five parameters: r_0 (the scale radius), v_0 , W_0 , \mathcal{R} (defined as the ratio of central densities $\rho_{0,dm}/\rho_{0,\star}$) and μ_{dm}/μ_\star . The μ_j have a physical interpretation when multicomponent models are applied to stellar mass classes. For our purposes, μ_{dm}/μ_\star determines the ratio between the King core radii of dark matter and stars, β . More specifically, $r_{c,j}^2 = r_0^2/\mu_j$ and thus $\beta \equiv \hat{r}_{dm}/\hat{r}_\star = (\mu_\star/\mu_{dm})^{1/2}$ (Gunn & Griffin 1979).

The distribution function has the form $f = \sum_j f_j$, implying that the dynamical-friction force is the linear sum of the contributions of stars and dark particles. Since the problem of the decay of GCs concerns those that are not far from the centre, we will focus on the behaviour of the volume density and velocity dispersion of stars and dark particles only within a distance $\sim 2\hat{r}_\star$ from the galaxy centre. This greatly simplifies the discussion and physical interpretation.

Under Eq. (3) it is easy to see that, within $\sim 2\hat{r}_\star$, the velocity dispersion profiles of the stellar and dark matter components are flat as long as $W_0 > 6$ and $\beta > 1$. This can be seen from very basic grounds. If $\beta = 1$, stars and dark particles are isothermal, both of which have the same velocity dispersion, within $2\hat{r}_\star$, for $W_0 > 6$ (see Fig. 4-11 of Binney & Tremaine 1987). If $\beta > 1$, the velocity dispersion profiles start to decline at larger radial distances (e.g., Pryor & Kormendy 1990; Walker et al. 2005). The reason is that $e^{-\mu_j(E-v_0^2 W_0)/v_0^2}$ is large compared to unity along a more extended radius range. In fact, as commonly argued, the flat behaviour of the stellar velocity profile in many dSph galaxies arise naturally if the stars orbit inside a dark matter halo with a King core radius larger than that of the visible component ($\beta > 1$). If, in addition, each component satisfies $\mu_j W_0 > 6$, the central velocity dispersion of each component j is simply related to v_0 according to $\sigma_j^2 = \mu_j^{-1} v_0^2$ and

$$\sigma_j^2 = \frac{4\pi G}{9} \rho_0 r_j^2, \quad (4)$$

where $\rho_0 = \rho_{0,dm} + \rho_{0,\star}$ (Gunn & Griffin 1979). The best fit dynamical two-component King models in Fornax suggest $7 < W_0 < 9$, with still larger values not being ruled out (Walker et al. 2005). Models with $W_0 < 7$ are excluded. Hence, the condition $\mu_\star W_0 > 6$ is fulfilled, bearing in mind that the more concentrated component (the stars in our case) has $\mu_j > 1$.[‡]

Unfortunately, this family of models presents serious limitations because the assumption that the luminous and dark components are dynamically coupled does not permit to have large values of β . In a recent careful analysis of stellar velocity dispersions in Fornax using two-component King models, Walker et al. (2005) found that the models able to reproduce the data span a narrow range in size of the dark halo, $2 < \beta < 3$. As discussed in detail by Walker et al. (2005), the restriction $\beta < 3$ is an artifact of the two-component King model, since for large β it produces excessively steep luminous matter profiles (Pryor & Kormendy 1989).

Since our goal is to obtain how the DF timescale depends on β , we will adopt a different approach. We will assume a certain form for the density profile of the stars and dark matter and derive the velocity dispersion through a Jeans equation:

$$\sigma_j^2(r) = \frac{\rho_{0,j} \sigma_j^2(0)}{\rho_j(r)} - \frac{1}{\rho_j(r)} \int_0^r \rho_j(r') \frac{G(M_{dm}(r') + M_\star(r'))}{r'^2} dr', \quad (5)$$

[‡] We may exploit the fact that the velocity dispersions σ_{dm} and σ_\star are constant within $r < 2\hat{r}_\star$, to find a relation between the dark matter and the stellar density profile $\rho_{dm} = \mathcal{R}(\rho_\star/\rho_{0,\star})^\lambda$, where $\lambda = \sigma_\star^2/\sigma_{dm}^2 \simeq \hat{r}_\star^2/\hat{r}_{dm}^2$.

where $M_*(r)$ and $M_{\text{dm}}(r)$ are the mass of stars and dark matter within radius r , respectively. Because the King model is known to provide a good fit to the present light distribution of the inner regions of certain dSph, globular clusters and large galactic dark haloes (Hernandez & Gilmore 1998a) and to keep matters simple, we will assume that the distributions of both the luminous component and the dark matter follow a King profile, being the dark matter halo more extended than the population of stars, so that \hat{r}_{dm} will be larger than \hat{r}_* . With the present quality of the data, we found that values $2 < \beta < 5$ are consistent with the line-of-sight velocity dispersion profile measured in Fornax. Other profiles for the dark halo and their implications will be explored in §3.4.

For obvious reasons, there have been many attempts to find an analytical fit to the luminosity profile of GCs and dSph galaxies: the modified Hubble law (1930), the empirical King profile (King 1962) or the Plummer model (Lake 1990) are some examples. In this section we present two analytical approximations which yield good approximate fits to King density profiles. The first is:

$$\rho(x) = \frac{\pi\rho_0}{4} \left(\frac{\text{erf}(x)}{x} \right)^2, \quad (6)$$

where $x \equiv 3r/2\hat{r}$, ρ_0 the central density and $\text{erf}(x)$ is the error function, and the second:

$$\rho(x) = \frac{\pi\rho_0}{4F} \left(\frac{\text{erf}(x)}{x} \right)^2, \quad (7)$$

where F is a correction factor equal to 1.0 for $r/\hat{r} < 2.0$ and equal to $2 - (2\hat{r}/r)$ for $r/\hat{r} > 2.0$.

Figure 1 gives the logarithm of the ratio of the density profiles of Eq. (6) and Eq. (7) to the exact King profiles, for the eight values of the shape parameters 2, 4, 6, 8, 10, 12, 14 and 16, left to right. It can be seen that for all shape parameters larger than or equal to 4, Eq. (6) is an extremely accurate fit to the King profile within one core radius. We see also that Eq. (6) is an excellent approximation to within a factor of 1.06 ($\log = 0.025$) of the exact value, internal to two core radii, for all shape parameters larger than or equal to 8. This accurately takes into account the drop in density observed within the core radius of a King sphere, of a factor close to 0.5. It should be noted that this drop is much more substantial than the corresponding one in the velocity dispersion, which remains flat to within a few percent inside the core region (Binney & Tremaine 1987).

For shape parameters between 12 and 14, Eq. (7) lies within a factor of 1.04 of the exact King profile within 25 core radii. The two equations above, together with Figure 1, are included as they might serve as accurate approximations to full King profiles in related work and are useful to derive analytical estimates. Our problem is restricted to the core region of the dark matter distribution, so that in the following only Eq. (6) will be used.

3.1 Dynamical friction in a two-component system

Under the approximation that the GCs orbit at the circular speed $v_c(r)$ as they spiral to the centre, the stellar component contributes to the DF force with:

$$F_*(r) = -\frac{4\pi \ln \Lambda G^2 \rho_*(r) M_p^2}{v_c^2} \left[\text{erf} \left(\frac{v_c}{\sqrt{2}\sigma_*} \right) - \sqrt{\frac{2}{\pi}} \frac{v_c}{\sigma_*} \exp \left(-\frac{v_c^2}{2\sigma_*^2} \right) \right], \quad (8)$$

where $\sigma_*(r)$ can be obtained from Eq. (5) and the circular velocity is given by

$$v_c(r) = \sqrt{\frac{G(M_*(r) + M_{\text{dm}}(r))}{r}}. \quad (9)$$

An analogous form for the contribution of the background of dark matter particles, $F_{\text{dm}}(r)$, can be derived by replacing $\rho_* \leftrightarrow \rho_{\text{dm}}$ and $\sigma_* \leftrightarrow \sigma_{\text{dm}}$. Notice that in this approach there is no guarantee that the distribution function is strictly Maxwellian. However, the errors made in this approximation are likely small.

As said before, we will use the profile given by Eq. (6) for the underlying stellar population, with central density $\rho_{0,*}$, as well as for the dark matter, with central density $\rho_{0,\text{dm}}$. Taking this analytical density profile, which is as accurate as a factor of 1.06 in the interval $0 < r < 2\hat{r}$, provided that the shape parameter > 8 , ensures that we incur in errors much smaller than the measured uncertainties in, for example, the stellar core radius or the inferred central matter density for Fornax.

For simplicity and to isolate the relevant physics, we will take the Coulomb logarithm constant with a value $\ln \Lambda = 3$, regardless of whether the interaction occurs with dark particles or stars. This is indeed an approximation since Λ is expected to be a linear function of the distance of the GC to the centre of the galaxy, $\Lambda \sim r/b_{\min}$ (e.g., Hashimoto et al. 2003; Just & Peñarrubia 2005). Therefore, our calculations overestimate the drag at small radii. Note, however, that the absolute value of Λ near the stellar core radius may vary in the range 3–4 depending on the half-mass radius of the GCs (see §2).

Once we know the friction force we can solve numerically the orbital evolution of a GC. Before presenting the results, we will give an estimate of the characteristic DF timescale for a GC within the core.

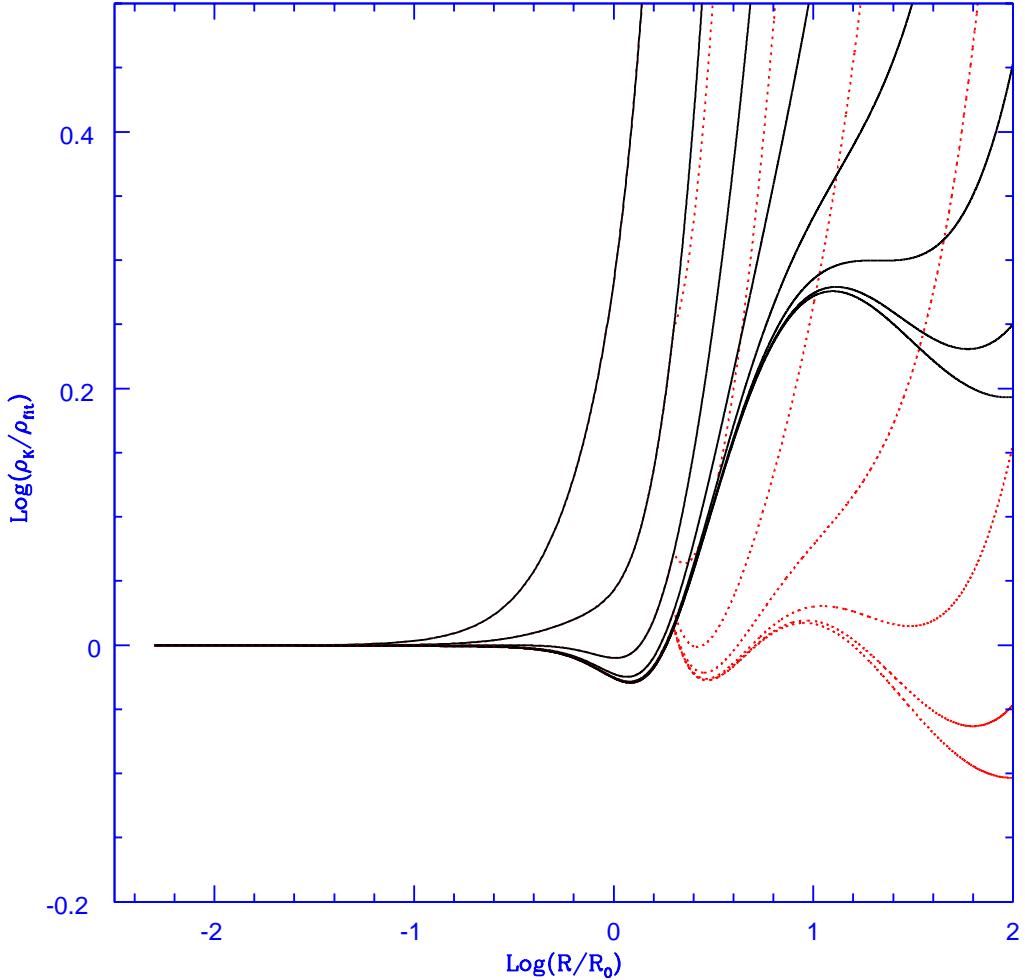


Figure 1. Logarithms of the ratios of the analytical density profiles as given in Eqs (6) and (7) to exact King profiles (solid and dotted lines, respectively) with shape parameters of 2, 4, 6, 8, 10, 12, 14 and 16 (from left to right).

3.2 Characteristic two-component orbit-decay timescale

It is convenient to gain additional insight by examining the drag felt by a GC bound to orbit within the stellar core radius, where $v_c^2 \approx 4\pi G(\rho_{0,*} + \rho_{0,dm})r^2/3$. Substituting into Eq. (8) and using $\sigma_*^2 = 4\pi G\rho_{0,*}(1 + \mathcal{R})\hat{r}_*^2/9$ as derived in the exact two-component King model (see Eq. 4), the DF force by the stars can be written as:

$$F_*(r) = -\frac{3 \ln \Lambda GM_p^2}{(1 + \mathcal{R})r^2} \left[\operatorname{erf} \left(\sqrt{\frac{3}{2}} \frac{r}{\hat{r}_*} \right) - \sqrt{\frac{6}{\pi}} \frac{r}{\hat{r}_*} \exp \left(-\frac{3r^2}{2\hat{r}_*^2} \right) \right]. \quad (10)$$

From Eq. (10) we see that the stellar contribution to the friction force is reduced by a factor $1 + \mathcal{R}$, as compared to the force if all the matter were concentrated in one single component following the light distribution. In the same approximation but for the dark matter component, we obtain:

$$F_{dm}(r) \simeq -\frac{3 \ln \Lambda GM_p^2}{(1 + \mathcal{R}^{-1})r^2} \left[\operatorname{erf} \left(\sqrt{\frac{3}{2}} \frac{r}{\hat{r}_{dm}} \right) - \sqrt{\frac{6}{\pi}} \frac{r}{\hat{r}_{dm}} \exp \left(-\frac{3r^2}{2\hat{r}_{dm}^2} \right) \right]. \quad (11)$$

For analytical purposes, we have used here the simplifying assumption that in dark matter dominated systems, $\sigma_{dm}^2 \simeq 4\pi G\rho_{0,dm}\hat{r}_{dm}^2/9$, which is certainly a lower limit. After a Taylor expansion of Equation (11) for $r \ll \hat{r}_{dm}$, we find

$$F_{dm}(r) = -3\sqrt{\frac{6}{\pi}} \frac{\ln \Lambda GM_p^2}{1 + \mathcal{R}^{-1}} \frac{r}{\hat{r}_{dm}^3}. \quad (12)$$

This shows that for relatively large values of \hat{r}_{dm} , corresponding to high σ_{dm} , the friction force by the dark matter component may be suppressed considerably, leaving a door open for the survival of the orbits of GCs. The reduction of F_{dm} in extended haloes was already highlighted by Hernandez & Gilmore (1998b) and Lotz et al. (2001).

Within the luminous core of the dwarf galaxy, the total friction force is:

$$F(r) = F_{\star}(r) + F_{\text{dm}}(r) \simeq -3\sqrt{\frac{6 \ln \Lambda}{\pi}} \frac{GM_p^2}{\hat{r}_{\star}^3} \left[1 + \frac{\mathcal{R}}{\beta^3} \right]. \quad (13)$$

As a measure of the rate of the orbit decay, we define τ_{5th} as the time for the GC orbit to decay to 1/5th of its initial radius. The calculation of the characteristic timescale τ_{5th} is similar to that set out in Binney & Tremaine (1987). As such, we only give the final result:

$$\tau_{5th} \simeq 0.77 \left[\frac{1 + \mathcal{R}}{1 + \mathcal{R}/\beta^3} \right] \frac{v_c(\hat{r}_{\star}) \hat{r}_{\star}^2}{GM_p \ln \Lambda} \simeq 1.33 \left[\frac{1 + \mathcal{R}}{1 + \mathcal{R}/\beta^3} \right] \frac{\sigma_{\star} \hat{r}_{\star}^2}{GM_p \ln \Lambda}. \quad (14)$$

The previous formula constitutes the generalization of Eq. (2) for the two-component case, provided that the object resides well inside the stellar core radius. For values $\mathcal{R} \gtrsim 10$ and $\beta \gtrsim 2$, the dynamical friction timescale can be significantly enhanced by a factor $\gtrsim 5$. In particular, for $\mathcal{R} = 10$, $\beta = 2$ and the above mentioned reference values of $\sigma_{\star} = 11 \text{ km s}^{-1}$, $\hat{r}_{\star} = 0.6 \text{ kpc}$ and $M_p = 2 \times 10^5 \text{ M}_{\odot}$ we get $\tau_{5th} \simeq 10 \text{ Gyr}$. We see that without violating stellar velocity dispersion constraints, dynamical friction timescales can be significantly extended by assuming cored dark matter haloes. By varying the King core radius of the dark halo in the range $2 < \beta < 5$, we are changing the dark matter particle distribution function, increasing their assumed velocity dispersion and hence suppressing their dynamical friction contribution, but the effect on the dark matter bulk gravitational force contribution within the halo core, responsible for the observed stellar kinematics, is small enough to be entirely compatible with present quality of the data. In the following we present the exact evolution of the orbit for a GC in a dwarf spheroidal galaxy with a mass and size similar to those of Fornax.

Returning to the opening discussion of §2, dynamical friction estimates based on Eq. (2), taking the measured stellar core radii $\sim \hat{r}_{\star}$ and velocity dispersions σ_{\star} , implicitly assume that mass follows light, and thus the friction drag with the dark matter component is included. Therefore, even though Eq. (2) can be written in terms of the stellar parameters only, it does not follow that the drag caused by the stellar wake alone yields excessively short DF timescales.

3.3 Numerical results

As shown in the previous section, we expect that for values of \mathcal{R} and β large enough, the problem of the coalescence of GCs in the central region of the host galaxy should be alleviated. Following the description given in §3.1, the evolution of the orbit of a GC was derived for a plausible range of parameters. We present the orbital decay for different values of \mathcal{R} , β and the circular velocity of the total mass distribution at \hat{r}_{\star} , denoted by \hat{v}_c , keeping the stellar core radius fixed at the 0.6 kpc measured for Fornax. In fact, the set $(\mathcal{R}, \beta, \hat{v}_c, \hat{r}_{\star} = 0.6 \text{ kpc})$ determines the mass model completely. The velocity dispersion and \hat{v}_c are related through $\hat{v}_c \sim \sqrt{3}\sigma_{\star}$. Thus, observations of σ_{\star} provide an estimate of \hat{v}_c , implying $\hat{v}_c \sim 20 \text{ km s}^{-1}$ for the observed value $\sigma_{\star} \approx 11 \text{ km s}^{-1}$ in Fornax. The halo parameters for our calculations were chosen to span the observational estimates of the mass of Fornax. Lokas (2002) derived a $M/L = 15\text{--}25$ inside a galactocentric radius of 2 kpc by modeling the moments of the line-of-sight velocity distributions. Using projected positions and radial velocities of stars in Fornax, Wang et al. (2005) estimated a $M/L = 7\text{--}22$ within a radius of 1.5 kpc, and Walker et al. (2005) suggest a cumulative $M/L = 7\text{--}10$ within the core radius. Hence, $\mathcal{R} \sim 7\text{--}20$ is consistent with observations. These authors also show that Fornax contains an extended dark halo; we will explore values for \hat{r}_{dm} in a reasonable range of $\sim 1\text{--}2 \text{ kpc}$, which implies $\beta = 1.6\text{--}3.2$.

The evolution of the orbital radius of a GC of mass $2 \times 10^5 \text{ M}_{\odot}$ moving initially on a circular orbit, for different mass models, is shown in Fig. 2. Given the linear nature of Newtonian gravity, as seen in the preceding sections, the dynamical friction force will be the sum of that due to stars and dark matter, c.f. Eq. (13). Still, these two components are coupled through the self-consistent galactic mass model, as the stellar velocity dispersion is largely determined by the dark matter component. Considering the large overall mass to light ratios of dSph systems, one might naively think that dynamical friction is determined completely by the F_{dm} term in Eq. (13), and that the effect of stars can be ignored. However, the details of local density and different velocity dispersion of these two components, and the local rather than global nature of dynamical friction make things more complicated. In order to illustrate this issue, we have separated both contributions in panel (a) of Fig. 2. The curve labeled with ‘‘stars’’ represents the radial evolution if the dark halo acts as an unresponsive external potential so that it does not produce any dynamical friction, i.e. $F_{\text{dm}} = 0$. The curve ‘‘dark matter’’ was calculated with $F_{\star} = 0$. In both cases $\mathcal{R} = 7$, $\hat{v}_c = 20 \text{ km s}^{-1}$ and $\beta = 2.5$ (or, equivalently, a King core radius $\hat{r}_{\text{dm}} = 1.5 \text{ kpc}$). It is interesting to note that even though the density of dark matter is a factor ~ 7 larger than the baryonic density, both drag forces are comparable for a GC traveling within the stellar core; both components must be included if an accurate description of the problem is to be obtained.

The evolution when F_{dm} and F_{\star} are both included is plotted in panel (b), but now for a GC starting at a distance $1.5\hat{r}_{\star}$ in

order to explore a larger dynamical range in radius. We can see that in this case the tendency of sedimentation of the clusters is significantly suppressed as compared to the one-component estimate (Eq. 2). In fact, a massive object of mass $2 \times 10^5 M_\odot$ initially at $1.5\hat{r}_*$ takes 10 Gyr to reach the stellar core radius and more than 25 Gyr to decay its radius to 1/5th of its initial value. If the same object is initially located at the stellar core radius, it would sink to a radius $0.2\hat{r}_*$ in 20 Gyr approximately. Roughly speaking, for the mass model under consideration ($\mathcal{R} = 7$, $\beta = 2.5$ and $\hat{v}_c = 20 \text{ km s}^{-1}$), a GC embedded in the stellar core will reduce its orbital radius by only a factor 2 during its lifetime. With these parameters the problem of the dramatic decay of the orbit is overcome.

For comparison and to show the sensitivity of the evolution to changes on β , let us compare the orbital evolution between a cluster in a model $\mathcal{R} = 7$, $\beta = 2.5$ and $\hat{v}_c = 15 \text{ km s}^{-1}$ (dashed-line in Fig. 2b), and in a model $\mathcal{R} = 7$, $\beta = 1.7$ and $\hat{v}_c = 20 \text{ km s}^{-1}$ (Fig. 2c). For the first model, a cluster initially at the stellar core radius will take 7.7 Gyr to shrink its radius a factor 2, whereas in the second one, this requires only 5 Gyr. For the latter model a noticeable radial contraction of the population of GCs as a whole is expected, which is not observed in Fornax.

The following sets of parameters satisfy the condition that all the clusters that are initially at a spatial radius $> 1.5\hat{r}_*$ from the centre of the host dwarf galaxy, remain outside a sphere of radius $0.75\hat{r}_*$ after 10 Gyr: for a core radius $\hat{r}_{\text{dm}} = 1.0 \text{ kpc}$, we need $\mathcal{R} = 15$ and $\hat{v}_c = 25 \text{ km s}^{-1}$, whereas for $\hat{r}_{\text{dm}} = 1.5 \text{ kpc}$, a value $\mathcal{R} = 7$ and velocity $\hat{v}_c = 15 \text{ km s}^{-1}$ suffice. The simplest hypothesis for the GC population is that they started out with the same spatial distribution as the underlying stellar population. However, the initial spatial distribution of GCs is unknown and could have been more diffuse than at present. Still, even for a GC population initially distributed as the stars, GCs could have avoided sedimentation toward the core of Fornax for a range of halo parameters, as noted above. Of course, the evolution of the radial distribution of a typical dSph or dE globular cluster system requires Monte Carlo simulations in order to include the mass function of the globular clusters which could depend in principle on the radius (see Lotz et al. 2001).

It is worthwhile to emphasize here that the mass models with $\hat{v}_c \sim 20 \text{ km s}^{-1}$ are consistent with the observed stellar kinematics, but values $\hat{v}_c \sim 25 \text{ km s}^{-1}$ are marginally inconsistent. In Fig. 3, the circular velocity of the halo for the mass model $\mathcal{R} = 7$, $\beta = 2.5$ and $\hat{v}_c = 15 \text{ km s}^{-1}$, is shown. The total mass-to-luminosity ratio interior to 2 kpc is 20 for this model. A comparison with the models of Lokas (2002) and Walker et al. (2005) reveals that this model is fully compatible with the observed kinematics of the stars (see also Kazantzidis et al. 2004). In fact, Walker et al. (2005) found that two-component King models can reproduce the radial velocity of the new sample of stars belonging to Fornax. For these models, the cumulative mass-to-light ratio within the core radius is 7–10 (see their Fig. 9). Since we are using two-component models similar to those used by Walker et al. (2005), we do not repeat the calculations of the expected velocity dispersion profile here.

In conclusion, the preservation of the clusters is assured if Fornax has a cored dark halo with physically reasonable King core radii of $\hat{r}_{\text{dm}} \sim 1.5 \text{ kpc}$ and $\mathcal{R} \gtrsim 7$. This mass model implies a central velocity dispersion for dark matter particles of $\sim 30 \text{ km s}^{-1}$ and a total mass $\sim 1.2 \times 10^9 M_\odot$. These parameters are also plausible for other dSph and dE (see §3.6). Notice that a very general result of galactic formation scenarios is that typical extents of the dissipationless dark matter halo are expected to be larger than those of the baryonic component, which collapse into the bottom of the potential well by dissipative processes, by factors of $\gtrsim 10$ (e.g. Fall & Efstathiou 1980; Kregel et al. 2005).

3.4 Other profiles for the dark halo: Distinguishing cores from cuspy haloes

In the previous section we have demonstrated that the solution to the GC decay problem may reside in a halo with a core somewhat larger than that of the stellar population. It is therefore natural to check whether a cored halo is a necessary condition or if cuspy haloes have the same interesting potential. In this section we will show that clusters cannot avoid sinking into the central region of Fornax if the dark halo is cuspy. Therefore, in the lack of any viable mechanism responsible for the dynamical heating of the GC system, the requirement of their survival may serve as a useful tool to discriminate between cuspy or cored haloes. In §3.6 other observational constraints regarding the dark matter haloes of dSph are given. Before doing so, it is convenient to consider the DF timescale for GCs embedded in dark haloes with density profiles different from the King spheres considered so far. When comparing dark matter profiles with different functional forms, we shall do so at fixed observational consequences, i.e., requiring final galactic models having similar velocity dispersion profiles for the stars. We will see that cored haloes are favored against cuspy haloes in dSphs.

We use a broad family of density profiles for the halo:

$$\rho(r) = \frac{\rho_s}{(r/r_s)^\alpha [1 + (r/r_s)^\gamma]^\beta}. \quad (15)$$

The parameters (α, β, γ) determine the shape of the density profile. The mass density distribution in the inner parts is described by a power-law $\rho \sim r^{-\alpha}$. The set of parameters $(0, 1, 2)$ corresponds to the pseudo-isothermal profile. Our aim here is to find out the friction timescale for cuspy haloes, say $\alpha > 0$. For this purpose we consider two broadly used profiles: the singular isothermal sphere $(2, 0, \gamma)$, and the NFW profile $(1, 2, 1)$, suggested by cosmological N-body simulations, e.g. Navarro et al. (1996). The profile of the singular isothermal sphere can be also reproduced adopting $(0, 1, 2)$ in the limit $r_s \rightarrow 0$ and

$\rho_s r_s \rightarrow \text{constant}$. That is, the singular sphere can be recovered as the limit of a pseudo-isothermal sphere with a vanishing core radius. As we have seen in §3.3 that a core with a minimum size is needed to explain the present configuration of GCs, we can anticipate that the singular isothermal sphere will predict an excessively short dynamical-friction timescale.

In the singular isothermal halo with density $\rho(r) = v_c^2/4\pi G r^2$, where v_c is the circular velocity, the time for the orbit of a globular cluster to decay from an initial radius r_i to the centre is

$$t_{\text{df}} = \frac{2.64 \times 10^2}{\ln \Lambda} \left(\frac{r_i}{2 \text{kpc}} \right)^2 \left(\frac{v_c}{250 \text{ km s}^{-1}} \right) \left(\frac{10^6 \text{ M}_\odot}{M_p} \right) \text{ Gyr} \quad (16)$$

(e.g., Binney & Tremaine 1987). We see that, in contrast to the cored halo case, where the orbital radius decay exponentially in time, in the singular isothermal sphere, as well as in a NFW halo (Taffoni et al. 2003), the body experiencing dynamical friction reaches the centre in a finite time. This must be borne in mind when comparing the timescales in this section to what was obtained for King spheres. Note, however, that the difference between t_{df} and $\tau_{5\text{th}}$ is not quantitative important in these models because the DF timescale is determined mainly by the initial orbital radius where the density is smallest. In fact, $\tau_{5\text{th}} = 0.96 t_{\text{df}}$ in the singular isothermal sphere.

The previous formula has proved very useful in the study of decay timescales of massive objects in the outer parts of the halo where the main contribution to the dynamical friction comes from dark matter particles following a $\rho \sim r^{-2}$ profile. Let us evaluate Eq. (16) assuming that the halo of Fornax follows the singular isothermal sphere at any radius. We need to estimate the associated circular velocity v_c for this halo. From the mass models of Lokas (2002) with varying index α and stellar anisotropy, it turns out that a singular isothermal halo with a value v_c larger than 30 km s^{-1} would be in conflict with the observed kinematics of the stars in Fornax, since it overestimates the stellar velocity dispersion. Hence, if generously adopting it as a characteristic value of v_c , Equation (16) with $\ln \Lambda = 3$ predicts an uncomfortably short timescale $t_{\text{df}} \lesssim 4.5 \text{ Gyr}$ for a GC of mass $2 \times 10^5 \text{ M}_\odot$ and initially at a distance $r_i = 0.6 \text{ kpc}$, to reach the centre caused by the dynamical friction with the dark matter particles alone. Only those GCs $r_i > 1.5 \text{ kpc}$ are expected to avoid dramatic sinking towards the centre in one Hubble time. In order for GCs placed within the core of the dSph to survive for 10 Gyr, one should invoke extremely large values $v_c > 65 \text{ km s}^{-1}$, which is inconsistent with the data.

The NFW density profile, proposed as a universal fitting formula for CDM haloes in the hierarchical clustering scenario (Navarro et al. 1996, 1997), is less cusped than the singular isothermal sphere, and has the form:

$$\rho(r) = \frac{\rho_s r_s^3}{r(r + r_s)^2}, \quad (17)$$

where the parameters ρ_s and r_s are the density scale and scale radius, respectively. Taffoni et al. (2003) provided an expression for t_{df} , assuming a NFW profile for the dark halo:

$$t_{\text{df}} \simeq 2 \frac{V_{200} r_i^2}{G M_p \ln \Lambda}, \quad (18)$$

where r_i is the initial radius and V_{200} is the circular velocity at r_{200} , which encompasses a mean overdensity of 200 times the critical density. By modeling the moments of the line-of-light velocity distribution in Fornax, Lokas (2001) obtained a best-fitting virial mass in the range $M_{200} = 1.3\text{--}1.5 \times 10^9 \text{ M}_\odot$, depending on the degree of anisotropy assumed for the stellar orbits, for a concentration parameter $c \approx 20$, as suggested in cosmological simulations. The specification of the halo mass and concentration allows all other parameters to be deduced. In particular, we get $V_{200} \simeq 17.5 \text{ km s}^{-1}$, and hence, Eq. (18) predicts $t_{\text{df}} \approx 4.5 \text{ Gyr}$ for a GC of mass $2 \times 10^5 \text{ M}_\odot$ and $r_i = 0.6 \text{ kpc}$ (we continue using $\ln \Lambda = 3$).

Bearing in mind that the observational determinations of core radii and velocity dispersion profiles of dSph galaxies are substantially larger than the few percent error incurred by the use of the King approximations, we see that their use does not affect our conclusions. In essence, the result is that changing the assumed dark matter profile from cored to cusped, within restrictions imposed by stellar kinematics, the DF timescales change by a factor of $\gtrsim 3$, allowing one to exclude the latter option, in spite of the approximations of the analysis or the uncertainties in the data.

3.5 Other galaxies

Fornax is not the only nearby dwarf galaxy with a population of GCs. Five GCs were identified in the dwarf elliptical satellite of Andromeda, NGC 185 (Hodge 1974). Since the stellar core radius of NGC 185 is only 0.1 kpc , Tremaine (1976) derived a decay rate four times faster than in Fornax. The explanation of an extended dark halo for the problem of the survival of those GCs may be problematic as the value of the mass-to-light ratio for this galaxy of ~ 3 is rather low (Held et al. 1992) and, hence, the interaction with the stars rather than with the dark matter will dominate the drag, with no much room left to dilute the effect of the friction (see the two-component formula, Eq. 13). However, the closest GC to the centre of this galaxy is at a projected distance of $\sim 6r_* \sim 0.6 \text{ kpc}$, well outside the core of the galaxy. NGC 185 is a clear case where the globular cluster system is much more extended than the stellar population. At such distances from the centre, the stellar density drops by a large factor making dynamical friction inefficient.

Lotz et al. (2001) examined a sample of dE galaxies for evidence of dynamical friction in their globular cluster systems and nuclear properties. They concluded that for the fainter galaxies, some mechanism should be working against the orbital decay of GCs: mass loss via supernova-driven winds, large dark matter cores greater than ~ 2 kpc, the formation of new star clusters, or tidal stripping. Our calculations give further support to the suggestion that dark matter King core radii greater than ~ 2 kpc for the fainter dE galaxies can explain the observed trend.

3.6 Cores in dwarf galactic haloes

In the previous sections, the study of dynamical friction timescales in dwarf galactic systems has lead us to conclusions supporting the idea of constant density cores for the dark matter haloes of such galaxies. In the present subsection we discuss alternative and independent lines of evidence pointing in the same direction.

Observations of rotation curves from the Milky Way to nearby dwarf galaxies and low surface brightness galaxies generally show that their dark matter density profiles have a flat inner core (e.g., Binney & Evans 2001; van den Bosch et al. 2000; de Blok & Bosma 2002; de Blok 2005). This appears to be in disagreement with the results of numerical simulations of CDM haloes. Despite the ongoing debate about the correct interpretation of the gas rotation curves of low-mass disc galaxies (Hayashi et al. 2004; Spekkens et al. 2005), cored profiles appear to be favoured over cuspy halo models (e.g., Gentile et al. 2005). The gas-free Local Group dSph galaxies provide an independent test for dark matter models as they are often completely dark matter dominated at all radii, and it is therefore possible to measure their dark matter content by treating their baryonic content as a massless tracer population. If dSph obey the same scaling relation $\rho_{0,\text{dm}}$ vs \hat{r}_{dm} found empirically by Fuchs & Mielke (2004) in low surface brightness galaxies, a core radius of ~ 3 kpc should be expected for a central density as that inferred for Fornax-like dSph, $\sim 0.02 \text{ M}_\odot \text{ pc}^{-3}$.

A great effort has been made in order to constrain the total M/L in dSph. The density profile of the haloes and their parameters still remain very uncertain, mainly due to large uncertainties in both the velocity dispersion measurements and the anisotropy of the stellar velocity dispersion. Lokas (2002, 2003) studied a set of dark matter profiles with different slopes in the inner parts and showed that all dark matter profiles yield good fits but only profiles with cores are consistent with isotropic orbits. A *King core radius* of ~ 1.5 kpc required to preserve the GC configuration in Fornax, as inferred in §3.3, can fit reasonably well the stellar velocity dispersion profile observed in Fornax (e.g., Lokas 2002; Stoehr et al. 2002; Kazantzidis et al. 2004; Walker et al. 2005). Throughout this paper, by core radius \hat{r} we always mean the radius at which the projected density falls to half of its central value. In cosmological simulations, it is usual to define the core radius $r_{c,\gamma}$ as the point where the density profile becomes steeper than γ . In order to avoid confusion between the various scale radii, the following relations are useful when comparing with cosmological models of structure formation: $r_{c,-1} \approx 0.75\hat{r}$ and $r_{c,-0.1} \approx 0.2\hat{r}$.

There exist other independent lines of evidence for cores in dSph. The recent identification of a kinetically cold stellar substructure in the Ursa Minor dSph (Kleyna et al. 2003) strongly suggests that the dark halo of that galaxy has a central core. Using N-body simulations Kleyna et al. (2003) showed that this cold structure would survive for less than 1 Gyr if the dark halo is cusped. Only if the dark halo has a uniform density core can the cold substructure survive. Additionally, Magorrian (2003) found an inner slope $\alpha = 0.55 \pm 0.35$ for the Draco dSph.

If it is confirmed that dSph galaxies contain cored haloes, then the problem of the formation of cores becomes universal in dwarfs, implying that cores are not exclusive of disc galaxies where any conflict between predictions and observations can be attributed to our poor understanding of the complicated physics of baryons, such as feedback, pressure support or transport of angular momentum by bars. This fact would have serious implications for models aimed to explain cores. dSphs with extended dark haloes also arise in Λ CDM simulations (Stoehr et al. 2002), the question to ask is how these extended haloes transform their central cusps into cores.

Although the formation of cores is a delicate issue, some possible ways have been explored in the literature. Sánchez-Salcedo (2003) proposed that decaying dark matter could explain the formation of cores in LSB and dSph. Read & Gilmore (2005) have suggested that the sudden impulsive loss of baryonic mass in a dSph could produce a redistribution of matter, transforming a central density cusp into a near-constant density core. Jin et al. (2005) show that a halo which is composed of massive black holes and which initially has a NFW density profile, can be transformed into a cored halo through dynamical evolution. The core radius of these haloes is ~ 1 kpc and resemble Draco's halo.

The present study puts constraints on the epoch of core formation. In fact, if cored haloes are solely responsible for the survival of the present-day GC configuration, as suggested in this paper, the formation of the core should have proceeded in an early phase of the galaxy's formation, at least 6 Gyr ago, to prevent excessive drag of Fornax GCs.

4 THE ALTERNATIVE: MOND

4.1 MOND in dSph and dynamical friction

An interesting alternative theory to the existence of massive dark matter haloes is a modification of the standard Newtonian gravity for accelerations below some characteristic value, $a_0 = 1-2 \times 10^{-8} \text{ cm s}^{-1}$ (Milgrom 1983). The success of MOND in reproducing the observed velocity rotation curves of spiral galaxies without dark matter haloes is amazing, only in about 10% of the roughly 100 galaxies considered in the context of MOND does the predicted rotation curve differ significantly from that observed (e.g., Sanders & McGaugh 2002). Although there are some aspects where MOND predictions are not fully satisfactory in disc galaxies (e.g., Sánchez-Salcedo & Hidalgo-Gámez 1999; Blais-Ouellette et al. 2001; Binney 2003; Gentile et al. 2005; Sánchez-Salcedo & Lora 2005), MOND remains an intriguing alternative to dark matter at galactic scales. In principle, the parameter a_0 should be universal and, having determined its magnitude, one is not allowed to leave it as a free parameter. However, the derived value depends upon the assumed distance scale. Here, the MOND acceleration parameter is assumed to be the value derived in Begeman et al. (1991), rescaled to the new distance scale, i.e. $a_0 = 0.9 \times 10^{-8} \text{ cm s}^{-2}$ (Bottema et al. 2002).

Because of their large sizes, dSph galaxies also lie in the regime of small accelerations and hence, provide a laboratory to test MOND. In the previous section we show that the survival of GCs in dSph may provide new insight into the halo properties. We will consider now the implications for MOND. Ciotti & Binney (2004) have already pointed out that the existence of GCs in dwarfs may be a serious difficulty for MOND. In order to address the severity of this potential problem, we wish to compare the timescale of GC orbital decay in MOND to that in the dark matter scenario in the case study of Fornax, a dSph galaxy with well-determined photometric (structural) parameters, as a diagnostic of MOND.

Lokas (2002) has applied MOND to Fornax, Draco and Ursa Minor dwarfs. She found that if the stellar-to-mass ratio is fixed to $1 \text{ M}_\odot/\text{L}_\odot$, the best fitting values of a_0 , which is a universal constant in MOND, lie in the range acceptable to explain the rotation curves of spirals, but much higher in the case of Draco and Ursa Minor. Conversely, adopting a value $a_0 = 1.2 \times 10^{-8} \text{ cm s}^{-2}$, the best fitting model for Draco corresponds to a $M/L = 32 \pm 10 \text{ M}_\odot/\text{L}_\odot$. Therefore, Draco needs a dark component even in MOND (see also Lokas, Mamon & Prada 2005), weakening the reliability of the MOND paradigm, but this fact by itself does not rule out MOND[§]. In Fornax, the best-fitting model requires a reasonable stellar-to-mass ratio of $1.8 \pm 0.4 \text{ M}_\odot/\text{L}_\odot$ and no dark matter at all. Since we are primarily interested in the GCs evolution in the context of Fornax, we will adopt in our study the stellar-to mass ratio within this range.

In isolated spherical systems, the nonlinear MOND field equation for the gravitational potential reduces to an algebraic relation between the real acceleration \mathbf{g} and the Newtonian acceleration \mathbf{g}_N , $\mu(|\mathbf{g}|/a_0)\mathbf{g} = \mathbf{g}_N$ (Brada & Milgrom 1995), where $\mu(x)$ is a smooth function which is not specified, but approaches 1 in the limit of large x and approaches x in the limit of small x (the deep MOND regime). It can be seen that the acceleration $g = v_c^2/r$ felt by a star in the core of Fornax is significantly smaller than a_0 and thus, for the dynamics of an object in Fornax the deep MOND regime would apply. Because of MOND's nonlinearity, a system's internal dynamics can be altered by an external field of acceleration g_{ext} within which it is immersed (Bekenstein & Milgrom 1984). A measure of the external field effect in a dwarf at a position D from the parent galaxy is the parameter $\eta \equiv 1.5(\sigma_*/V)^2(D/\hat{r}_*)$ (Milgrom 1995), with V the galactic rotational velocity at D , which coincides with the asymptotic rotation velocity V_∞ for all the dwarfs. In the quasi-Newtonian limit, i.e. when $\eta \ll 1$ and all accelerations relevant to the dwarf dynamics are smaller than a_0 , the dynamics becomes Newtonian but with a larger effective gravitational constant of Ga_0/g_{ext} . Hence, we need to know the degree of isolation of the dwarf to investigate the dynamics of GCs. Note also that given the luminosity profile and the stellar M/L , the circular velocity curve may be different for the isolated case and the quasi-Newtonian case.

By considering general principles of statistical mechanics, Ciotti & Binney (2004) found that the dynamical-friction time in an isolated system in the weak acceleration limit, is shorter by a factor $a_0^2/(\sqrt{2}g^2)$ over the value it would have in a Newtonian system with the same stellar mass and a fixed auxiliary gravitational potential. The reason for this is that encounters at impact parameters comparable to the half-mass radius are dominant and provide the major contribution to the DF in MOND. The approach developed by Ciotti & Binney can be extended easily to the quasi-Newtonian limit if the calculation is carried over with the substitution $g \rightarrow g_{\text{ext}}$ and hence, the DF time in the quasi-Newtonian limit is shorter than in the Newtonian case by a factor $a_0^2/(\sqrt{2}g_{\text{ext}}^2)$.

In Fornax $\eta = 0.85$ and therefore it is a borderline case as regards MOND isolation. Neither limit is valid in this case but the two limits should give similar values for the friction. In the next section we will explore two extreme situations: Fornax in isolation and Fornax in the quasi-Newtonian limit. Since the force law depends on the degree of isolation, it is important to keep in mind that the stellar mass-to-light ratio required to fit the observed velocity dispersion profile is not the same in both limits. Hence, once given the stellar kinematics and the luminosity profile, the mass density is different in the isolated

[§] Note that there is consensus in the discrepancy between the baryonic mass and the dynamical mass in clusters of galaxies in MOND.

weak-field limit and in the quasi-Newtonian regime. Hereafter, where appropriate, the scripts ISO will denote the isolated regime and QN the quasi-Newtonian limit.

In the derivation of the friction force acting on a GC at a distance from the centre of the dwarf r , we will adopt the local approximation. That is, we extend the results of Ciotti & Binney (2004) evaluating the force with the local variables. The local approximation assumes that there are no strong gradients in the properties of the stellar background. However, it is liable to objection because of the long-range behaviour of the MOND law. In Newtonian dynamics much effort has been done to prove that the local approximation is a realistic assumption for most astrophysical scenarios. In the context of MOND this remains to be checked. Therefore, although our results should be taken with caution, the DF force based on the local approximation might have uncertainties of as much as a factor 2 in a cored dSph.

4.2 MOND Orbital-decay timescale in galaxy cores - Isolated galaxy under deep MOND regime

As said in the previous subsection, in the framework of deep-MOND regime, the DF force exerted by the stars on a massive object orbiting within the bulk of an isolated dwarf is enhanced by a factor $a_0^2/(\sqrt{2}g^2) = a_0^2r^2/(\sqrt{2}v_c^4)$ over its Newtonian value. Hence, the DF force at a radius r from the centre of the dwarf in the local approximation is:

$$F_*^{\text{ISO}}(r) = -\frac{4\pi \ln \Lambda G^2 \rho_*(r) M_p^2}{v_c^2} \left(\frac{r^2 a_0^2}{\sqrt{2}v_c^4} \right) \left[\text{erf} \left(\frac{v_c}{\sqrt{2}\sigma_*} \right) - \sqrt{\frac{2}{\pi}} \frac{v_c}{\sigma_*} \exp \left(-\frac{v_c^2}{2\sigma_*^2} \right) \right]. \quad (19)$$

In the deep MOND limit, the circular velocity is given by $v_c(r) = (Ga_0 M_*(r))^{1/4}$, and the mean one-dimensional velocity dispersion by $\sigma_*^4 = (4/81)Ga_0 M_T$ (Bekenstein & Milgrom 1984), where M_T is the total (baryonic) mass. Substituting the expression for $v_c(r)$, with $M_*(r) \simeq 4\pi\rho_0 r^3/3$, into Eq. (19), we find that massive objects in the inner core, i.e. at those radii such as $M_* \ll M_T$, spiral towards the dwarf centre at a constant rate according to:

$$\frac{dr}{dt} = -0.45 \frac{Ga_0 M_p \ln \Lambda}{\sigma_*^3}, \quad (20)$$

where local circularity of the orbit has been assumed. The dynamical-friction timescale for MOND of a body initially at circular orbit with radius r_i is then

$$\tau_{5th}^{\text{ISO}} = 1.8 \frac{r_i \sigma_*^3}{Ga_0 M_p \ln \Lambda}. \quad (21)$$

Starting at a radius inside the core, say 0.3 kpc, this DF timescale is extremely short, ~ 0.09 Gyr, for our fiducial values $\sigma_* = 11 \text{ km s}^{-1}$, $M_p = 2 \times 10^5 \text{ M}_\odot$ and $a_0 = 0.9 \times 10^{-8} \text{ cm s}^{-2}$. As noticed first by Ciotti & Binney (2004), the DF timescale is of the order of the dynamical crossing time. In fact, the orbital evolution is so fast that our assumption that the decay is through circular orbits is not fully justified. In the next section, however, we solve numerically the orbit of a massive GC in Fornax, without demanding local circularity. For a given σ_* taken from observations of a certain dSph, since τ_{5th} is inversely proportional to a_0 , our adopted scaled value of a_0 , which is smaller than others previously considered in the literature (e.g., The & White 1987), gives a generous estimate of the DF timescale.

4.3 Dwarf galaxy in an external field in the quasi-Newtonian limit

In the quasi-Newtonian limit, the form of the force is similar to the Newtonian case but replacing $G \rightarrow Ga_0/g_{\text{ext}}$, plus an extra-factor of $\sqrt{2}$ (see §4.1):

$$F_*^{\text{QN}}(r) = -\frac{4\pi \ln \Lambda G^2 \rho_*(r) M_p^2}{v_c^2} \left(\frac{a_0^2}{\sqrt{2}g_{\text{ext}}^2} \right) \left[\text{erf} \left(\frac{v_c}{\sqrt{2}\sigma_*} \right) - \sqrt{\frac{2}{\pi}} \frac{v_c}{\sigma_*} \exp \left(-\frac{v_c^2}{2\sigma_*^2} \right) \right]. \quad (22)$$

A similar analysis to the Newtonian one carries over to get a formula for τ_{5th}^{QN} for bodies orbiting well inside the core:

$$\tau_{5th}^{\text{QN}} = 1.88 \left(\frac{g_{\text{ext}}}{a_0} \right) \left(\frac{\sigma_* \hat{r}_*^2}{GM_p \ln \Lambda} \right), \quad (23)$$

resulting in an expression formally identical to Eq. (14), with $\mathcal{R} = 0$, except for the factor $\sqrt{2}(g_{\text{ext}}/a_0)$. For Fornax standard values, $\tau_{5th}^{\text{QN}} = 0.3$ Gyr, i.e. a factor $\sim (V_\infty/\sigma_*)^2 (\hat{r}_*/D) (\hat{r}_*/r_i) = 1.5\eta^{-1} (\hat{r}_*/r_i) = 1.7 (\hat{r}_*/r_i)$ larger than in the isolated case. However, we warn that these timescales were derived for an object in the inner core of a galaxy and, therefore, it is not clear at this stage whether this difference between the timescales in the isolated and quasi-Newtonian cases also holds for a body placed initially outside the core radius. In fact, let us estimate the ratio between the magnitudes of the local DF force in the isolated case (Eq. 19) and in the quasi-Newtonian case (Eq. 22) at a certain radius r . For simplicity of the discussion, suppose that in both cases the circular velocities v_c at r are roughly the same, which implies that the M/L must be rearranged to have similar stellar kinematics, i.e. similar σ_* , in both limits. After some manipulations, it is easy to show that the ratio of the forces is given by:

$$\frac{F_*^{\text{QN}}}{F_*^{\text{ISO}}} = \frac{v_c^2}{r g_{\text{ext}}} = \left(\frac{v_c}{V_\infty} \right)^2 \left(\frac{D}{r} \right). \quad (24)$$

For a typical Fornax-like galaxy, the characteristic circular velocity in Fornax is 25 km s^{-1} at $1.5\hat{r}_*$. Equation (24) implies that F_*^{QN} is a factor ~ 2 larger than F_*^{ISO} at $1.5\hat{r}_*$. In fact, we will show in the next section that the spiraling of a GC initially at $1.5\hat{r}_*$ is faster in the quasi-Newtonian limit than in the isolated weak-field limit, in a Fornax-like galaxy with $\eta \sim 1$.

4.4 Determination of the orbital evolution in MOND

Using Eq. (6) as an approximation for the density profile of Fornax, and assuming that the stellar orbits are isotropic with a roughly constant velocity dispersion σ_* , the orbital evolution of a GC of mass $2 \times 10^5 \text{ M}_\odot$ has been integrated numerically in the local approximation in both regimes discussed above: isolated weak field limit and quasi-Newtonian limit. In principle, since in the MOND paradigm there is no dark matter, the stellar mass-to-light ratio is the only free parameter and thus, it also fixes the value of σ_* . However, in order to explore the sensitivity of the results on the velocity dispersion, the orbit was calculated taking σ_* as an independent parameter spanning the reasonable range of $10\text{--}15 \text{ km s}^{-1}$.

Figure 4 shows the in-spiral of the GC sinking because of the DF due to the stars only, for different stellar M/L ratios. As Eq. (21) suggests, the decay rate in the isolated regime depends strongly on the adopted value for σ_* . In this regime, the GC reaches 1/5th the initial radius in less than ~ 4 orbits. The GC evolves maintaining its orbit circular until it reaches $\sim 0.5\hat{r}_*$ (see Fig. 4b).

A comparison of the curves labeled by B_{QN} and B_{ISO} reveals that the in-spiral takes a longer time in the isolated limit than in the quasi-Newtonian limit. As already said, the stellar M/L are different in order to have comparable kinematics (i.e. similar rotation curves within $1.5\hat{r}_*$) in both cases. The time required for a GC initially at $1.5\hat{r}_*$ to reach the $0.3\hat{r}_*$ is 1.6 times larger in the isolated weak acceleration limit than in the quasi-Newtonian regime. Note however, that the in-spiral within $0.3\hat{r}_*$ is faster in the isolated case in accordance with our discussion in §4.3. Indeed, the decay in the QN limit is exponential at small radii (see Fig. 4).

The slowest in-spiral corresponds to the case with the highest σ_* and M/L parameters. Notice that a ratio $M/L = 3.2$ lies outside 2σ the value quoted by Lokas (2002) in Fornax under MOND. But even if adopting these parameters for MOND, a massive GC initially in circular orbit at $1.5\hat{r}_*$ reaches the centre of the dwarf in ~ 2 Gyr. Hence, MOND predicts a complete sedimentation of the GCs of Fornax. In fact, without adopting *ad hoc* conditions, MOND is unable to be compatible with the observed possession of GCs by Fornax. Consequently, one should invoke external heating effects to rescue MOND. However, the effects of this heating mechanism should be observable as distortions in the photometry, for which, as mentioned previously (§2), there is no evidence.

The possession of GC by dwarf galaxies (dE and dSph) is likely challenging for the MOND theory. One could ameliorate the problem of the orbital decay just by decreasing the universal acceleration a_0 and adding a classical dark halo. But one needs to reduce a_0 up to a value $\sim 10^{-9} \text{ cm s}^{-2}$ for which the classical Newtonian dynamics at galactic scales is recovered. This solution has no astrophysical interest because then a dark component has to be added as well to explain the missing mass problem in spiral galaxies. This component will become the main explanation for the missing mass at galactic scales and not only at cosmological scales (Pointecouteau & Silk 2005).

5 CONCLUSIONS

Long before relevant data was available and using a simple but transparent scaling of the dynamical friction timescale, Tremaine (1976) warned that nuclei should be formed in the centre of dSph galaxies by coalescence of GCs. He noticed that NGC 185, NGC 147 and Fornax all have GCs but no nucleus. More recently, and for a sample of 51 dwarf elliptical galaxies in nearby clusters, Lotz et al. (2001) examined the radial distribution of GCs and found that for the fainter dE's, the predicted nuclei was several magnitudes brighter than observed. Since the composite radial distribution of the GCs follows the exponential stellar profile of dE's at present day, the simplest hypothesis is that the GCs initially followed the exponential profile of the underlying stellar population and that the dynamical friction force was partly inhibited. Note that the drag should not be completely suppressed in order to explain the significant deficit of bright, massive GCs in the inner regions of the sample of Lotz et al. (2001). In Local Group dSph galaxies, the problem of the survival of enhanced density substructure with cold kinematics against dynamical friction has been revived recently (Kleyna et al. 2004; Walker et al. 2006). For the less massive galaxies, the tidal field could play a role in heating their GCs but they must be tidally disrupted and suffer significantly mass loss (50–90%) in order to counter the drag of dynamical friction. If so, these galaxies should present a rising projected velocity dispersion profile beyond a critical radius caused by tidal stripping (Read et al. 2006). In contrast, the Local Group dSph galaxies show a very flat or falling projected velocity dispersion, which indicates that tidal stripping is unimportant interior to ~ 1 kpc.

In this paper we have shown that all these features can be explained by considering dark haloes as having density cores.

Taking Fornax as a reference case, we have explored under which structural parameters of the dark halo is the orbital decay of GCs minimized (see also Goerdt et al. 2006 for an independent and complementary study). Using self-consistent dynamical models where both the stars and dark matter contribute to the frictional drag, we have computed the radial evolution of a massive GC initially on a circular orbit. We find that the contributions to dynamical friction of stars and dark matter are both comparable in dwarf galaxies. The survival, against dynamical friction, of GCs requires a King core radius for the dark halo slightly larger than that of the underlying stellar population. In the particular case of Fornax, we infer a dark halo with the following parameters: a King core radius of ~ 1.5 kpc (or $r_{c,-0.1} = 0.3$ kpc), a central velocity dispersion of ~ 30 km s $^{-1}$ and a total mass of 1.2×10^9 M $_{\odot}$. In a dark matter halo with a cuspy inner profile, the dynamical friction timescale is $\lesssim 4.5$ Gyr. The indirect evidence of a central density core in dSph has important implications for the formation of dSph galaxies and for cosmology, and gives new insight to the ongoing debate on the inner slope of the dark halo profiles in low surface brightness galaxies. The present study puts constraints on the epoch of core formation. If cored haloes are solely responsible for the survival of the present-day GC configuration, the formation of the core should have proceeded in an early phase of the galaxy's formation, at least 6 Gyr ago, to prevent excessive drag of Fornax GCs.

Under the MOND hypothesis, dynamical friction timescales for dwarf galaxies are necessarily short, ~ 1 Gyr. Therefore, alternative explanations must be found for the observed GC systems. No such entirely satisfactory alternative has been presented to date, making old GC systems in dwarfs a strong objection to MOND under its current formulation.

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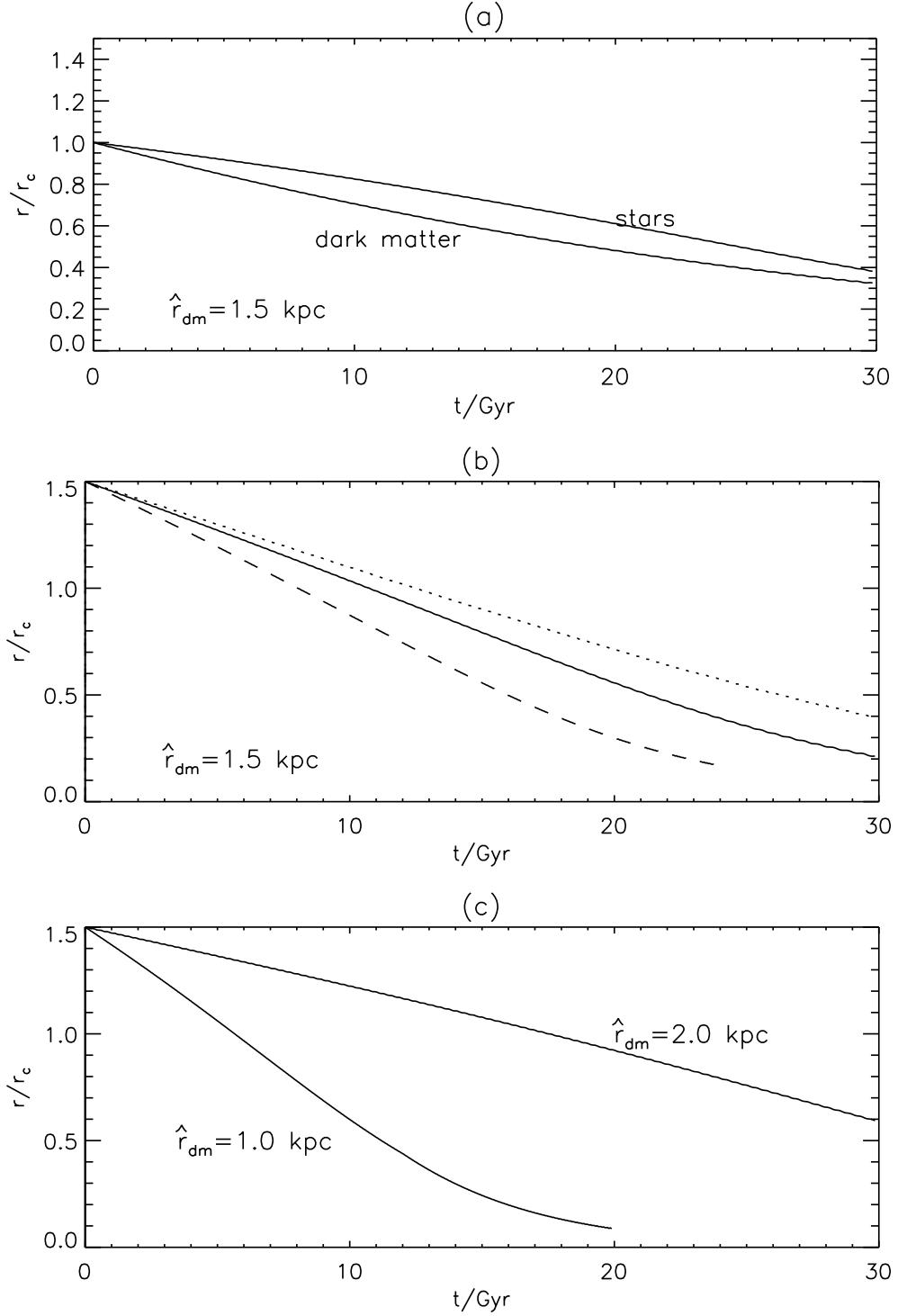


Figure 2. Evolution of the radius of the globular cluster for different parameters of the model. Orbital decay if only stars or dark matter contribute to dynamical friction for $\mathcal{R} = 7$ and $\hat{v}_c = 20 \text{ km s}^{-1}$ (panel a). Decay including both the stellar and dark matter components (panel b), for $(\mathcal{R}, \hat{v}_c) = (15, 20), (7, 20), (7, 15)$, from top to bottom. In panel (c) the dependence on the dark matter core radius is shown for $(\mathcal{R}, \hat{v}_c) = (7, 20)$.

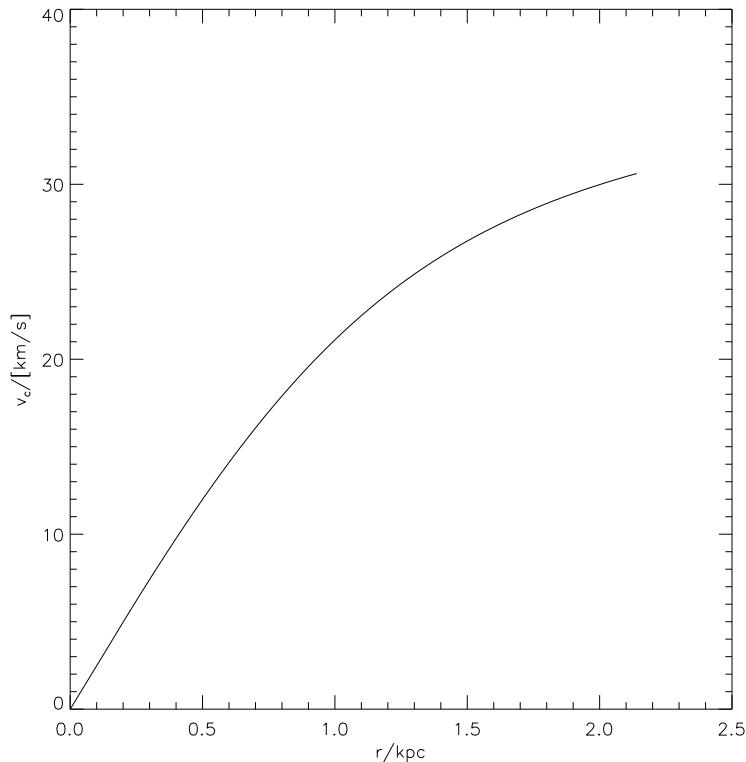


Figure 3. Circular velocity for the dark halo in the mass model $\hat{r}_{\text{dm}} = 1.5$ kpc, $\mathcal{R} = 7$ and $\hat{v}_c = 15$ km s $^{-1}$.

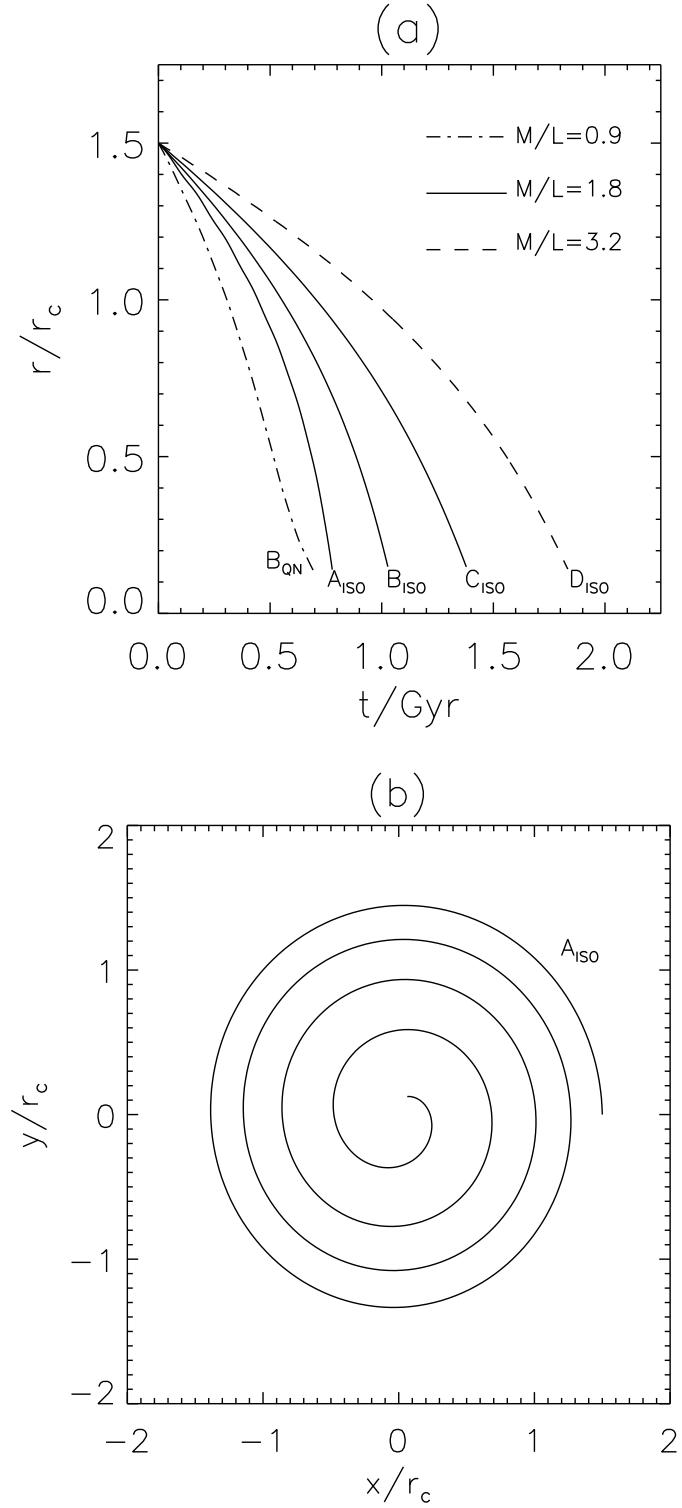


Figure 4. Panel (a): Temporal evolution of the radial position of a massive object of $2 \times 10^5 M_\odot$ in MOND for different values of the mass-to-light ratio and velocity dispersions $\sigma_* = 10, 12, 14$ and 15 km s^{-1} for curves A, B, C and D, respectively. Panel (b): Orbit in the (x, y) plane for case A.